Question	Scheme	Marks	AOs
1(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	……….and factorise $8k^3 \pm 12k^2 \pm 4k = 4k(2k^2 \pm 3k \pm 1) =$	dM1	1.1b
	Correct work with statement $4 \times ..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
			(5 marks)
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd × multiple of 4	A1	2.4
		(4)	

Notes:

(a)
Note: May be in any variable (condone use of *n*)

B1: Selects a correct strategy. E.g uses an odd number is $2k \pm 1$

M1: Attempts $(2k \pm 1)^3 - (2k \pm 1) = ...$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the *k* term or the constant of $(2k \pm 1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or 4*k* from their cubic

A1: Correct work with statement $4 \times ..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

Question	Scheme	Marks	AOs
2(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n\left(n+1\right)\left(n+2\right)$	M1	2.1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Notes

(i)

B1: Identifies the error in the statement by giving

- a counter example and a reason eg $x = -4$ with $x^2 = 16$ eg $x = -4$ with $(-4)^2 > 9$
- concludes **not true**

There should be no errors seen including the use of brackets. The conclusion could be a preamble. Do not accept "sometimes true" or equivalent.

Alternatively, explains why the statement is **not true**

Eg. It is not true as when $x < -3$ then $x^2 > 9$ so x does not have to be greater than 3.

Eg. $x^2 > 9 \Rightarrow x < -3$ or $x > 3$ so not true

(ii)

M1: Takes out a factor of n and attempts to factorise the resulting quadratic.

A1: Deduces that the expression is the product of 3 consecutive integers

A1: Explains that as the expression is a multiple of 3 **and** 2, it must be a multiple of 6 and so is divisible by 6

If you see any method which appears to be credit worthy but is not covered by the scheme then send to review

Question	Scheme	Marks	AOs
3 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
			(5 marks)

Notes:

(a)

B1: Provides a counter example with a reason. There is no need to state “not true”.

e.g., $7^3 - 2^3 = 335$ which divides by 5 {exactly}.

It is sufficient to have, e.g., $9^3 - 4^3 = 665$ and $\frac{665}{5} = 133$

Here q must be greater than p and both must be natural numbers, not 0 or negatives.
Note that any pair of positive integers n and $n+5k$ will provide a counter example, but $q^3 - p^3$ must be evaluated correctly, and if they divide by 5 this also needs to be correct.

(b)

M1: For the key step in stating the algebraic form of consecutive even numbers.
See main scheme for examples. They might be used either way round for this mark.

dM1: Attempts $(2n+2)^3 - (2n)^3 = ...$ condoning slips but must lead to a quadratic.

Alternatively, $(2n+2)^3 - (2n)^3 = 2^3\{(n+1)^3 - n^3\}$

May be subtracted the wrong way round for this mark as below.

$(2n)^3 - (2n+2)^3 = ...$ but this will score M1dM1A0A0

A1: e.g., $(2n+2)^3 - (2n)^3 = 24n^2 + 24n + 8$ **or** $(2n+4)^3 - (2n+2)^3 = 24n^2 + 72n + 56$

or $(2n+2)^3 - (2n)^3 = 8\{(n+1)^3 - n^3\}$ **or** $(2n)^3 - (2n-2)^3 = 24n^2 - 24n + 8$ etc.

Must come from correct work and the algebra will need checking carefully.

A1: For a full and rigorous proof showing all necessary steps including:

- correct quadratic expression for $q^3 - p^3$ for their even numbers, e.g., $24n^2 + 24n + 8$
- reason e.g., $24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ **or**, e.g., in $24n^2 + 24n + 8$ the coefficients are all multiples of 8
- minimal conclusion, "hence true"

Alt 1:

If the even numbers are set as n and $n + 2$ there must be sufficient work seen before marks can be awarded.
e.g.,

M1dM1: $n = 2k \Rightarrow (n+2)^3 - n^3 = ...n^2 + ...n + ... = ...(2k)^2 + ...(2k) + ...$

A1: $= 24k^2 + 24k + 8$

A1: $= 8(3k^2 + 3k + 1)$ so $q^3 - p^3$ is a multiple of 8

Alt 2:
If they just use any two even numbers, e.g., $2a$ and $2b$, **or** $2m$ and $2n + 2$ then they will score as follows:

M1: $(2a)^3 - (2b)^3$ Condone missing brackets if recovered.

dM1: $= ...a^3 - ...b^3$

A1: $= 8a^3 - 8b^3$ Note $8(a^3 - b^3)$ would imply this mark.

A1: $= 8(a^3 - b^3)$ so $q^3 - p^3$ is a multiple of 8 if q and p are {any two} even {numbers}

and hence $q^3 - p^3$ is a multiple of 8 if q and p are *consecutive* even numbers

Question	Scheme	Marks	AOs
4(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=...$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

Notes

(i)

M1: A full and rigorous argument that uses all of n = 1, 2, 3 and 4 in an attempt to prove the given result. Award for attempts at both $(n+1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27>9$

Extra values, say $n=0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for n =1, 2, 3 and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept ✓ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m=2p\pm1$ and attempting to expand $(2p\pm1)^3+5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p+1)^3+5=8p^3+12p^2+6p+6$ and **states** even.

Alternatively reaches $(2p-1)^3+5=8p^3-12p^2+6p+4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) **A reason** why the expression $8p^3+12p^2+6p+6$ or $8p^3-12p^2+6p+4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3-12p^2+6p+4=2(4p^3-6p^2+3p+2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if m^3+5 is odd then m is even"
- "this is contradiction, so proven."
- "So if m^3+5 is odd them m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a counter example to the statement "if m^3+5 is odd then m must be even" such as when $m=\sqrt[3]{2}$ then they can score special case mark B1

Question	Scheme	Marks	AOs
5	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **or** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	M1 A1	3.1a 2.2a
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n(\in □)$ *	A1*	2.4
		(4 marks)	

Notes

General guidance

It is likely that you will see a mixture of methods and approaches within some solutions.

Mark the approach which scores the highest number of marks.

There should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".
Withhold the final mark if n is used instead of k or reference to $n \in □$ but $n \in □^+$ is acceptable.

Main scheme algebraic method using e.g. $n=2k$ **and** $n=2k\pm1$
You will need to look at both cases and mark the one which is fully correct first.
Allow a different variable to k and may be different letters for odd and even

M1: For the key step attempting to find $(n+1)^3-n^3$ when $n=2k$ **or** $n=2k\pm1$ and attempting to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$ **or** $2n\pm5$)
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Complete argument for $n=2k$ **or** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd.
Requires:

- Correct simplified quadratic expression e.g. $12k^2+6k+1$ (when $n=2k$), $12k^2+18k+7$ (when $n=2k+1$), $12k^2-6k+1$ (when $n=2k-1$) (may be factorised)
- A reason why the expression is odd e.g. $2k(6k+3)+1$ or may use a divisibility argument e.g. $\frac{12k^2+6k+1}{2}=6k^2+3k+\frac{1}{2}$
- Concludes "odd" o.e. (may be within their final conclusion)
There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered"
Condone the use of e.g. $n=2n$ and $n=2n\pm1$

dM1: Attempts to find $(n+1)^3-n^3$ when $n=2k$ **and** $n=2k\pm1$ and attempts to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$, $2n\pm5$
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1*: Complete argument for **both** $n = 2k$ **and** $n = 2k+1$ (or e.g. $n = 2k-1$) showing the result is odd for all $n\left(\in \mathbb{N}\right)$

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. "Hence odd for all $n\left(\in \mathbb{N}\right)$" Accept "hence proven", "statement proved", "QED"

The conclusion for when $n = 2k$ and $n = 2k+1$ may be within the final conclusion rather than separate which is acceptable e.g. "when $n = 2k$ and when $n = 2k+1$ the expression is odd, hence proven" (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3 - n^3$
$n = 2k-1$	$8k^3$	$8k^3 - 12k^2 + 6k - 1$	$12k^2 - 6k + 1$
$n = 2k$	$8k^3 + 12k^2 + 6k + 1$	$8k^3$	$12k^2 + 6k + 1$
$n = 2k+1$	$8k^3 + 24k^2 + 24k + 8$	$8k^3 + 12k^2 + 6k + 1$	$12k^2 + 18k + 7$

Alternative methods:

Algebraic with logic example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.

A1: Correct quadratic expression $3n^2 + 3n + 1$

dM1: Attempts to factorise their quadratic such that $n^2 + n \to n(n+1)$ within their expression e.g. $3n(n+1)+1$

A1*: Explains that e.g. $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even so $3n(n+1)+1$ is odd hence odd for all $n\left(\in \mathbb{N}\right)$

Proof by contradiction example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.

A1: Correct quadratic expression $3n^2 + 3n + 1$

dM1: Sets $3n^2 + 3n + 1 = 2k$ (for some integer k) $\Rightarrow 3n(n+1) = 2k - 1$

A1*: Explains that $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even but $2k-1$ is odd hence we have a contradiction so $(n+1)^3 - n^3$ is odd (for all $n\left(\in \mathbb{N}\right)$. There must have been a correct opening statement setting up the contradiction e.g. "assume that there exists a value for n for which $(n+1)^3 - n^3$ is even"

Solutions via just logic (no algebraic manipulation)

e.g.

If n is odd, then $(n+1)^3 - n^3$ is $\text{even}^3 - \text{odd}^3 = \text{even} - \text{odd} = \text{odd}$

If n is even, then $(n+1)^3 - n^3$ is $\text{odd}^3 - \text{even}^3 = \text{odd} - \text{even} = \text{odd}$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n = k$, substitutes $n = k+1$ into $(n+1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2 + 9k + 7$ Condone arithmetical slips

A1: $\left(\mathrm{f}(k+1) = 3k^2 + 3k + 1 + 6(k+1) =\right)$ $(k+1)^3 - k^3 + 6(k+1) = \mathrm{f}(k) + 6(k+1)$
which is odd + even = odd

dM1: Attempts to substitute $n = 1 \Rightarrow (1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n = 1$
- if it is true for $n = k$ then it is true for $n = k+1$
- therefore it is true for all $n\left(\in \mathbb{N}\right)$